

we have conservation of mass

$$+ \frac{\partial \rho}{\partial t} = 0,$$

discussed the total momentum also

$$= 0, \quad (3)$$

slit width.

ish so on the left side of (1) and (2)
; (1) and (2) we get

$$\eta_n \nabla \times (\nabla \times \mathbf{v}_n) + (2\eta_n + \eta') \nabla (\nabla \cdot \mathbf{v}_n). \quad (4)$$

carried by the normal fluid such that

$$\mathbf{v}_n \beta^{-1} \quad (5)$$

to be conserved, $\nabla \cdot \mathbf{q} = 0$ and

$$\nabla \beta = \mathbf{q} \cdot \nabla \beta. \quad (6)$$

$$(\nabla \cdot \mathbf{v}) - \nabla^2 \mathbf{v}$$

simplified to give

$$- \rho_s (\mathbf{v}_s \cdot \nabla) \mathbf{v}_s - \rho_n (\mathbf{v}_n \cdot \nabla) \mathbf{v}_n. \quad (7)$$

me assumptions which will later be
ke the z axis along the length of the
ors in these directions are \mathbf{e}_z and \mathbf{e}_x .

$$T = T(z) \quad (8)$$

$\eta_n(z)$. We further assume that the
small compared to the other terms.
ted into x and z components to give²

$$\eta_n + \eta' \mathbf{q} \frac{d^2 \beta}{dz^2} \quad (9)$$

he heat current density appears are not
we continue to use the boldface notation

and

$$\frac{\partial P}{\partial x} = (\eta_n + \eta') \frac{d\mathbf{q}}{dx} \frac{d\beta}{dz}. \quad (10)$$

Equation (9) may be solved if we assume that the second term on the right is small, i.e.

$$\beta^{-1} \frac{d^2 \beta}{dz^2} \ll \frac{\eta_n}{2\eta_n + \eta'} \mathbf{q}^{-1} \frac{d^2 \mathbf{q}}{dx^2}. \quad (11)$$

(Justification for this assumption will be given later.) Subject to the condition $\mathbf{q} = 0$ at the slit boundaries $\pm d/2$, the solution for \mathbf{q} is

$$\mathbf{q} = \frac{3}{2} \bar{\mathbf{q}} \left(1 - \frac{4x^2}{d^2} \right). \quad (12)$$

Then the pressure gradient becomes

$$\frac{\partial P}{\partial z} = - \frac{12\eta_n \bar{\mathbf{q}}}{\rho s T d^2}. \quad (13)$$

This last equation is the basis of the so-called Allen-Reekie rule, which specifies that in the limit of small ΔT 's the fountain pressure P_f and the heat current density are proportional and that this relationship is independent of the form of \mathbf{F}_{sn} . Since the right hand side of (13) is strongly temperature dependent, for larger temperature differences this equation must be integrated to give

$$\Delta P_z = P_f = - \int_{T_0}^{T_1} \frac{12\eta_n \bar{\mathbf{q}}}{\rho s T d^2} \frac{dz}{dT} dT. \quad (14)$$

In order to obtain the relationship between P_f and $\bar{\mathbf{q}}$ for large temperature differences it is therefore necessary to obtain an expression for dT/dz as a function of the temperature along the length of the slit. Since the temperature gradient along the slit does depend upon \mathbf{F}_{sn} , as will be seen below, it is obvious that the relationship between P_f and $\bar{\mathbf{q}}$ must for large temperature differences also depend upon \mathbf{F}_{sn} .

We now wish to find an expression for the temperature gradient. To do so we must postulate a particular form for the frictional force \mathbf{F}_{sn} . We shall concentrate our attention upon the Gorter-Mellink type of force, which we shall write in the slightly generalized form

$$\begin{aligned} \mathbf{F}_{sn}(\mathbf{v}_s - \mathbf{v}_n) &= A \rho_s \rho_n (|\mathbf{v}_s - \mathbf{v}_n| - \mathbf{v}_c)^{m-1} (\mathbf{v}_s - \mathbf{v}_n) & |\mathbf{v}_s - \mathbf{v}_n| > \mathbf{v}_c \\ &= 0 & |\mathbf{v}_s - \mathbf{v}_n| < \mathbf{v}_c. \end{aligned} \quad (15)$$

Here A is the (temperature dependent) Gorter-Mellink coefficient, \mathbf{v}_c is a (possibly temperature dependent) critical velocity, and m has in various ex-