and

$$\frac{\partial P}{\partial x} = (\eta_n + \eta') \frac{d\mathbf{q}}{dx}\frac{d\beta}{dz}. \tag{10}$$

Equation (9) may be solved if we assume that the second term on the right is small, i.e.

$$\beta^{-1}\frac{d^2\beta}{dz^2} \ll \frac{\eta_n}{2\eta_n + \eta'}\, \mathbf{q}^{-1}\frac{d^2\mathbf{q}}{dx^2}. \tag{11}$$

(Justification for this assumption will be given later.) Subject to the condition $\mathbf{q} = 0$ at the slit boundaries $\pm d/2$, the solution for $\mathbf{q}$ is

$$\mathbf{q} = \frac{3}{2}\bar{\mathbf{q}}\left(1 - \frac{4x^2}{d^2}\right). \tag{12}$$

Then the pressure gradient becomes

$$\frac{\partial P}{\partial z} = -\frac{12\eta_n\bar{\mathbf{q}}}{\rho s T d^2}. \tag{13}$$

This last equation is the basis of the so-called Allen-Reekie rule, which specifies that in the limit of small ΔT's the fountain pressure P_f and the heat current density are proportional and that this relationship is independent of the form of $\mathbf{F}_{sn}$. Since the right hand side of (13) is strongly temperature dependent, for larger temperature differences this equation must be integrated to give

$$\Delta P_z = P_f = -\int_{T_0}^{T_1} \frac{12\eta_n\bar{\mathbf{q}}}{\rho s T d^2}\frac{dz}{dT}\,dT. \tag{14}$$

In order to obtain the relationship between P_f and $\bar{\mathbf{q}}$ for large temperature differences it is therefore necessary to obtain an expression for dT/dz as a function of the temperature along the length of the slit. Since the temperature gradient along the slit does depend upon $\mathbf{F}_{sn}$, as will be seen below, it is obvious that the relationship between P_f and $\bar{\mathbf{q}}$ must for large temperature differences also depend upon $\mathbf{F}_{sn}$.

We now wish to find an expression for the temperature gradient. To do so we must postulate a particular form for the frictional force $\mathbf{F}_{sn}$. We shall concentrate our attention upon the Gorter-Mellink type of force, which we shall write in the slightly generalized form

$$\begin{aligned}\mathbf{F}_{sn}(\mathbf{v}_s - \mathbf{v}_n) &= A\rho_s\rho_n(|\mathbf{v}_s - \mathbf{v}_n| - \mathbf{v}_c)^{m-1}(\mathbf{v}_s - \mathbf{v}_n) \qquad |\mathbf{v}_s - \mathbf{v}_n| > \mathbf{v}_c \\ &= 0 \qquad |\mathbf{v}_s - \mathbf{v}_n| < \mathbf{v}_c.\end{aligned} \tag{15}$$

Here A is the (temperature dependent) Gorter-Mellink coefficient, $\mathbf{v}_c$ is a (possibly temperature dependent) critical velocity, and m has in various ex-